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Forecasting Realised Volatility:
Does the LASSO approach outperform HAR?

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Abstract

The HAR model dominates current volatility forecasting. This model implies a restricted lag approach, with three parameters accounting for an AR(22) structure. This paper uses the Lasso method, which selects a parsimonious lag structure, while allowing both a flexible lag structure and lags greater than 22. In-sample results suggest that while significance is largely found among the first 22 lags, consistent with the HAR model, there is evidence that longer lags contain information, as Lasso models provide an improved fit. Out-of-sample forecasts for daily, weekly and monthly volatility, evaluated using MSE, QLIKE, MCS and VaR measures, suggest that the ordered Lasso model provides the preferred forecasts using an AR(100) at the daily level and an AR(22) for the weekly and monthly horizons. The results support the view that a more flexible lag structure is preferred over the HAR approach.

Keywords: Volatility Forecasting, HAR, Lasso, VaR
JEL Codes: C22, G12

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1. Introduction.

The HAR model of Corsi (2009) is the dominant approach to modelling and forecasting volatility of financial asset returns. Following the establishment of realised volatility (RV) as a method of obtaining a volatility series (a literature that largely began with Andersen and Bollerslev, 1998), the RV approach has overtaken the GARCH model as a way to examine volatility, culminating in the HAR model. The HAR model proposes a very specific lag structure, with lags aggregated to produce weekly and monthly variables, in addition to a daily lag. As Corsi (2009) notes, this model could be regarded as a restricted AR(22). This leads to two broad questions. First, could longer lags provide additional information that may improve forecasts? Second, is the restricted lag structure the most appropriate?

An alternative lag selection approach that is gaining traction within the field of finance is the Lasso (least absolute shrinkage and selection operator) modelling procedure proposed by Toshigami (1996). The Lasso approach produces a parsimonious parameter specification in a linear model, and can improve model efficiency (Friedman et al., 2010). The Lasso approach is increasingly employed in forecasting. For example, Audrino and Knaus (2016) extend the Lasso method used in the AR model of Nardi and Rinaldo (2011) to the HAR model. However, empirical results to date reveal limited, if any, improvement over the HAR model.

Therefore, this paper reconsiders the ability of the Lasso approach to outperform the HAR model in forecasting volatility for a range of international markets. As a baseline, we include both the HAR model of Corsi (2009) and the HAR-free extension (Bollerslev, et al., 2018), which allows the first six daily RV lags to be freely estimated. Nonetheless, both of these models use a fixed lag length approach as opposed to the flexible lag Lasso model. Thus, to answer the question of whether the fixed lag length is appropriate, we estimate an AR(22) model using the Lasso method. To examine the further question of whether longer lags may contain additional information, we consider an AR(100) (also used by Audrino and Knaus, 2016; Audrino et al., 2017). Furthermore, we include more recent developments in the Lasso literature that allows for different parameter penalty functions, including the adaptive Lasso (Zou, 2006), the grouped Lasso (Yuan and Lin, 2006), which is more suitable for strongly correlated variables, and the ordered Lasso (Toshigami and Suo, 2016).

To evaluate the performance of the alternative models, we conduct both rolling and recursive forecasts for eight international markets, and use both statistical (MSE, QLIKE and model confidence set) and economic (value-at-risk, VaR) based forecast evaluation. In preview of the results, we find that, in-sample, the Lasso based approaches provide a small improvement in model fit, while there is some evidence that lags longer than 22 do contain useful information. In the out-of-sample exercise, the Lasso approach significantly outperforms the HAR model in forecasting both volatility and VaR. Notably, the ordered Lasso with an AR(100) at the daily level and AR(22) at the weekly and monthly levels provide the best forecasts. The results presented here should be of interest not only to academics but also those actively engaged in risk management practice. They suggest that the Lasso flexible lag length approach can outperform the HAR model that currently dominates volatility forecasting.

2. Literature Review.

The analysis of time-varying behaviour within financial assets is important for asset allocation and risk management. Within the sphere of volatility modelling, it is well known that asset return volatility is persistent and can be captured using long-memory autoregressive type models (e.g., Ding et al., 1993; Bollerslev and Mikkelsen, 1996). While the initial impetus to volatility modelling came with the GARCH model (Engle, 1982; Bollerslev, 1986), Engle and Bollerslev (1986) first sought to capture long memory with the Integrated GARCH model. Subsequent developments include the Fractionally Integrated GARCH model of Baillie et al. (1996) and the Component GARCH model of Engle and Lee (1993). In addition, Granger and Ding (1996) discuss generalized fractionally integrated processes in a non-GARCH setting.

With the greater availability of intraday data, research increasingly focussed on the realised volatility (RV) approach of Andersen and Bollerslev (1998). Subsequent research examines time series models for RV and the long memory behaviour of high-frequency data (see, for example, Andersen et al., 2003; Lieberman and Philips, 2008; Martens et al., 2009). Following this, RV dominates the modelling and forecasting of volatility.

Anderson et al. (2004) suggest that an ARMA-type model for RV performs well in forecast exercises. While Hol and Koopman (2002) suggest that the long-memory feature of RV can be accommodated by an ARFIMA process, which provides more accurate predictions than a GARCH (and stochastic volatility, SV) model. However, some researchers question the fractional integrated process, Poskitt (2006) and Wang and Hsiao (2012) show that an ARFIMA(p, d, q) process can be well approximated by an AR(k) model using information criterion to determine the lag order. The empirical work of Wang et al. (2013) suggests that this AR-based method can provide better forecasting performance than a fractional integrated process. Alternatively, to accommodate RV long memory, Corsi (2009) proposes the HAR model. The HAR model has a simple autoregressive structure for RV with economically meaningful lagged average variables that represent different time horizons. As emphasized by Corsi (2009), the standard HAR model can be regarded as an unrestricted AR(22) model. Corsi et al. (2008) note that the forecasting ability of the HAR model outperforms fractional integrated models. Similar to other forecasting models, the standard HAR model can be extended to accommodate volatility jumps (Andersen et al., 2007) and sign asymmetry (or leverage; Narndorff-Nielsen et al., 2008; Corsi et al., 2012; Patton and Sheppard, 2015).

Despite the HAR model having an economically meaningful fixed lag structure (1, 5, 22, to represent daily, weekly and monthly time horizons), this fixed lag structure is questioned. Craioveanu and Hillebrand (2012) extend the fixed lag structure of the HAR model to a flexible lag structure, although they find no significant forecast improvement. Hwang and Shin (2014) extend the three lag HAR model to an infinite-order HAR model, i.e., HAR(∞), with exponentially decaying coefficients. However, the results show preference for the finite-order HAR(p) forecasts. An enhanced AR model with structural breaks is developed by Wand et al. (2013). This model outperforms ARFIMA-based methods in a forecast evaluation. The result of a change in the memory parameter provides an econometric explanation for the empirical success of the HAR model, which can be considered as a special case of the AR model.

The above empirical studies suggest that the lag structure of daily, weekly and monthly time horizon may be appropriate for forecasting volatility. However, the potential for an alternative lag structure exists. Toshigami (1996) proposes the least absolute shrinkage and selection operator (Lasso) to consider a larger lag structure. The Lasso process shrinks estimators towards exactly zero based on a tuning parameter. Empirical research using the Lasso approach is increasingly employed in different econometric settings. This includes Wang et al. (2007) and Hsu et al. (2008) in examining regression coefficients. For an AR model, Nardi and Rinaldo (2011) demonstrate that Lasso model selection, estimation and prediction is consistent under certain conditions. As the Lasso process provides parsimonious and efficient forecasting variables, it is increasingly used in a forecasting setting. For example, Li and Chen (2014), Roy et al. (2015) and Ziel (2016) demonstrate Lasso-based forecast superiority over other model specifications, while Tian et al. (2015) and Nazemi and Fabozzi (2018) both apply Lasso selected models to an asset modelling context. As noted, the Corsi (2009) HAR model is a restricted AR(22) with only three coefficients. Audrino and Knaus (2016) find the Lasso process of the same AR model has the same forecasting performance as the HAR model at the individual stock level. In other words, the fixed lag structure of the HAR model is hard to beat. However, the Lasso model cannot accurately restore the HAR lag structure, which raises questions about the suitability of the HAR lag structure (Audrino and Knaus, 2016).

In criticising the Lasso approach, Fan and Lin (2001) indicate potential inefficient and inconsistent model selection results. To address this, Zou (2006) allows more flexible penalization in obtaining estimators and proposes the adaptive Lasso, which uses an adaptive weight penalty to shrink the variable coefficients. Using the adaptive Lasso, Park and Sakaori (2013) and Audrino and Camponovo (2013) both indicate it provides more efficient estimators. In a robustness check in Audrino and Knaus (2016), the Lasso and adaptive Lasso have the same predictive ability as the HAR model. Subsequently, Audrino et al. (2019) consider the HAR model employing the adaptive Lasso method to test whether the lag structure of a flexible HAR model could recover the fixed HAR(1,5,22) model. They provide empirical evidence that shows only slight modelling outperformance for the flexible HAR model, with an insignificant out-of-sample difference. However, Fang et al. (2020) suggest using an adaptive Lasso can significantly improve the predictive ability of long-term volatility.

A common drawback of Lasso and adaptive Lasso is that they penalize every estimator separately and are not suitable for strongly correlated variables. As a consequence, in cases with correlated predictors, unreliable estimators are produced by Lasso and adaptive Lasso. However, in many multifactor regressions, variables are naturally grouped, as in the HAR model where lagged realised volatility is categorized into different time horizons. In addition, Hillebrand and Medeiros (2010) indicate that bagging lagged RV is reasonable and can improve foresting accuracy. Yuan and Lin (2006) therefore, propose the Group Lasso that considers group model selection and penalizes coefficients and selects estimators on a group, instead of an individual, basis. To check the validity of the lag structure for the HAR model, Audrino et al. (2019) use an AR(50) model and estimate by the Group Lasso method. However, the HAR lag structure is not supported and they conclude that there are lags that contain forecasting information beyond one month. Furthermore, there are two additional algorithms of Grouped Lasso. The first one is the Cluster Group Lasso by Buhlmann et al. (2013). Where variables are strongly correlated or have a near linear relation in a multifactor regression, the Cluster Group Lasso tends to choose only one variable from a group and neglects others. The second one is the Sparse Group Lasso (Friedman et al., 2010; Simon et al., 2013), in which the penalized parameter is employed at both group and individual level.

A further issue for the Lasso approach in capturing model dynamics, is the presence of both higher- and lower-order lags. Two models that allow lower-order lags to be considered with the inclusion of higher-order lags are the Hierarchical Lasso (Bien et al., 2013) and the Ordered Lasso (Toshigami and Suo, 2016). Both approaches focus on the selection of lower lagged coefficients before higher lagged coefficients. In empirical application, Wilms et al. (2016) indicate that the ordered Lasso is the best performing forecast for RV between these two Lasso approaches. They also argue that the ordered Lasso slightly outperforms the HAR model. Croux et al. (2018) employ the ordered Lasso model and find that it has better forecasting performance than the HAR model.

The current state of the literature suggests that the HAR model is preferred in forecasting volatility. However, an increasing amount of research questions the fixed lag length approach of the HAR model. To date, there is none, or at best limited, evidence that a more flexible lag structure can improve forecast performance. This paper addresses this question across a selection of eight international markets and a longer range of lags.

3. Methodology.

3.1. Empirical Models

As noted, the HAR model is established as the main volatility forecast model, we consider this model against Lasso-based alternatives, which allow for different lag specifications.

HAR Model

Following Andersen and Bollerslev (1998), we define day *t* realised volatility ($\mathrm{RV_t}$) as:

$$\mathrm{RV_t} = \sum_{\mathrm{i=1}}^{\mathrm{N}} \left(\mathrm{r_{t,i}}\right)^2 \quad (1),$$

where $r_{t,i}$ refers to the log asset return on day t and ith intraday interval and where $\mathrm{r_{t,i} = p_{t,i} - p_{t,i-1}}$, $(\mathrm{t} = 1, \cdots, \mathrm{T};\ \mathrm{i} = 1, \cdots, \mathrm{N})$, with T the total trading days, N the number of intraday intervals and $\mathrm{p_{t,i}}$ is the log asset price.

Corsi (2009) proposes the HAR model based on daily, weekly and monthly horizons, which correspond to different investing behaviour and is given as:

$$RV_t = \beta_0 + \beta_d RV_{t-1} + \beta_w RV_{t-1:t-5} + \beta_m RV_{t-1:t-22} + u_t \quad (2).$$

The HAR model is thus, a simple linear regression, where weekly and monthly averages of realised volatility are calculated as:

$$RV_{t-1:t-5} = \frac{1}{5}\sum_{i=1}^{5} RV_{t-i} \quad (3),$$

$$RV_{t-1:t-22} = \frac{1}{22}\sum_{i=1}^{22} RV_{t-i} \quad (4).$$

The HAR model thus models RV (volatility) as a linear equation of yesterday's RV and average RV over last week and last month. Corsi (2009) notes that the standard HAR model can be rewritten as an unrestricted AR(22) model:

$$RV_{t+1} = \theta_0 + \sum_{i=1}^{22} \theta_i RV_{t-i} + u_t \quad (5).$$

The restrictions on coefficients, θ_i, implied by the HAR lag structure are given as:

$$\theta_i = \begin{cases} \beta_d + \frac{1}{5}\beta_w + \frac{1}{22}\beta_m & for\ i = 1; \\ \frac{1}{5}\beta_w + \frac{1}{22}\beta_m & for\ i = 2, \dots, 5; \\ \frac{1}{22}\beta_m & for\ i = 6, \dots, 22. \end{cases} \quad (6).$$

The simplification from 22 parameters of an AR(22) to three parameters in the HAR model are empirically proven to improve model fit (Corsi, 2009). That said, the information criteria between the restricted HAR and the unrestricted AR(22) provides less clear cut results.

HAR-free model

To incorporate sudden unexpected volatility changes, Bollerslev et al. (2018) propose the HAR-free model as follow:

$$RV_t = \beta_0 + \beta_1 RV_{t-1} + \beta_2 RV_{t-2} + \beta_3 RV_{t-3} + \beta_4 RV_{t-4} + \beta_5 RV_{t-5} + \beta_6 RV_{t-6} + \beta_m RV_{t-1:t-22} + u_t \quad (7).$$

This augments the HAR model where the first six daily lagged RV are estimated freely, while $\beta_m RV_{t-1:t-22}$ is computed as in equation (4) above.

Lasso

Toshigami (1996) proposes the Lasso method, which, according to Friedman et al. (2010), can provide an efficient algorithm to select estimators that are computationally efficient. Recently, the Lasso method, and extensions, play an increasing role in econometrics and the forecasting of financial assets (Tian et al., 2015; Nazemi and Fabozzi, 2018).

The Lasso can be regarded as a constrained least square regression, where the tested model is a linear autoregressive one. The Lasso estimator of an AR(p) model is given as:

$$RV_{t+1} = \theta_0 + \sum_{i=1}^{n} \theta_i RV_{t-i+1} + u_t \quad (8),$$

where the Lasso estimator can be defined as:

$$\hat{\beta}_{lasso} = argmin\left\{\sum_{t=p}^{T}\left(RV_{t+1} - \theta_0 - \sum_{i=1}^{p}\beta_i RV_{t-j+1}\right)^2 + \lambda\sum_{i=1}^{p}|\beta_i|\right\} \quad (9),$$

where λ is the tuning parameter that controls the shrinkage estimators in term of penalty strictness. The first part of equation (9) is the least square criterion, and the second part is the penalty term on the regression parameters. Where $\lambda = 0$, the Lasso estimators will coincide with OLS estimators. Increasing λ causes more coefficients of the Lasso to be penalized to zero, with stricter coefficient selection. All the coefficients will be set to zero when $\lambda = 1$.

Adaptive Lasso

According to the basic Lasso method introduced by Toshigami (1996), every estimator is penalized equally. To address this, Zou (2006) develops the adaptive Lasso, which allows more flexible penalization to obtain estimators. The adaptive Lasso is given as follow:

$$\hat{\beta}_{adoptive\ lasso} = argmin\left\{\sum_{t=p}^{T}\left(RV_{t+1} - \theta_0 - \sum_{i=1}^{p}\beta_i RV_{t-j+1}\right)^2 + \lambda\sum_{i=1}^{p}\lambda_i|\beta_i|\right\} \quad (10),$$

where λ_i are adaptive weights for each coefficient. When every λ_i is equal to 1, the adaptive Lasso transforms into original Lasso. Compared with the standard Lasso, the adaptive Lasso allows a stricter penalty for zero coefficients and a lower penalty for non-zero coefficients. This reduces estimation bias and improves efficiency and accuracy of variable selection.

Group Lasso

A common drawback of Lasso and adaptive Lasso is that they penalize every estimator separately and ignore any correlation between each estimator. Thus, they can select one of the correlated estimators and omit others in the penalizing process. The group Lasso (Yuan and Lin, 2006) seeks to address this shortcoming. The group Lasso penalizes coefficients and selects estimators as a group instead of an individual variable and is given by:

$$\hat{\beta}_{group\ lasso} = argmin\left\{\sum_{t=p}^{T}\left(RV_{t+1} - \theta_0 - \sum_{i=1}^{p}\beta_i RV_{t-j+1}\right)^2 + \lambda\sum_{k=1}^{K}\sqrt{p_k}\sqrt{\sum_{i\in I_k}\beta_i{}^2}\right\} \quad (11).$$

Audrino et al. (2019) group an AR(50) as {1}, {2-5}, {6-22}, {23-50} and estimate using the group Lasso method. Noting that this lag structure does not perform well, they conclude that some lags have predictive information beyond the HAR lag structure. Using this information, the group lag structure chosen here includes the standard HAR approach of an AR(22) as {1}, {2-5}, {6-22} as well as longer groups based on a AR(100) as {23-50}, {51-75}, {76-100}.

Ordered Lasso

Toshigami and Suo (2016) argue that the predictive information of the coefficients should gradually decay. Thus, they introduce an autoregressive model within the Lasso approach with an additional monotonic decreasing constraint, the order-constrained coefficients. The ordered Lasso is given by:

$$\hat{\beta}_{ordered\ lasso} = argmin\left\{\sum_{t=p}^{T}\left(RV_{t+1} - \theta_0 - \sum_{i=1}^{p}(\beta_j^+ + \beta_j^-)RV_{t-j+1}\right)^2 + \lambda\sum_{i=1}^{p}(\beta_j^+ + \beta_j^-)\right\} \quad (12),$$

subject to $\beta_1^+ \geq \beta_2^+ \geq \cdots \geq \beta_p^+ \geq 0$ and $\beta_1^- \geq \beta_2^- \geq \cdots \geq \beta_p^- \geq 0$. The ordered Lasso model modifies the penalized parameters from the absolute value ($|\beta_j|$) of Lasso to positive and negative components ($\beta_j^+ + \beta_j^-$), allowing some β_j coefficients to be estimated as exactly zero. In addition, this ordered constraint penalty only allows higher lag orders to be estimated when lower lag order lags are already included.

3.2. Cross-Validation

A key element in the Lasso-method is tuning parameter (λ), which determines the flexibility of parameter estimation and the number of non-zero coefficients. Two common approaches to this in the literature are to use information criteria and cross-validation (CV).

Here, we choose λ based on the K-folds cross-validation method (Nardi and Rinadlo, 2011; Audrino et al., 2017; Audrino et al., 2019). Toshigami (1996) first estimates the prediction error of the Lasso approach using this method. The sample observations are split into K groups, denoted G_K, the estimators are obtained on $K-1$ groups and the test error is predicted on the remaining group. The process is repeated for $k = 1, 2, \cdots, K$, and the results of test error are averaged, with the procedure conducted for each value of the tuning parameter λ. We set $k = 10$ and the cross-validation error function is the mean square error (MSE), given as:

$$\text{CV}\ (\lambda) = \frac{1}{T}\sum_{k=1}^{K}\sum_{G_K}\left(y_t - \widehat{y_t^k}(x_t)\right)^2 \qquad (13),$$

where $\widehat{y_t^k}(x_t)$ is the prediction of K^th-fold. The optimal λ is selected by minimising the error of CV (λ):

$$\widehat{\lambda_{CV}} = \arg\min_{\lambda} CV(\lambda) \qquad (14).$$

Alternatively, the tuning parameter can be estimated by the AIC (Akaike) or BIC (Bayesian) Information Criterion. Audrino and Knaus (2016) determine λ by minimising the BIC, while Nardi and Rinaldo (2008) estimate an AR model using the AIC. Taking the analysis of Audrino and Knaus (2016) a step further, Wilms et al. (2016) and Croux et al. (2018) produce the tuning parameter λ with a forecast combination using a weighted BIC. In the empirical application, Wilms et al. (2016) note only slight improvement in performance compared to non-combination models. Wand et al. (2007) argue that a Lasso model with either CV or BIC produces the same fit. Audrino and Knaus (2016) and Audrino et al. (2019) also note that results are qualitatively similar across the alternative approaches.

3.3. Forecasting Evaluation

To evaluate and compare the accuracy of the different forecasting models, we follow Patton (2011) and use the Quasi-Likelihood (QLIKE) and Mean Squared Error (MSE) measures. These are robust to heteroscedasticity and given by:

$$\mathrm{MSE} = \frac{1}{\mathrm{n}}\sum_{\mathrm{t}=1}^{\mathrm{n}}(\mathrm{RV_t} - \widehat{\mathrm{RV}}_\mathrm{t})^2 \qquad (15),$$

$$\mathrm{QLIKE} = \frac{1}{\mathrm{n}}\sum_{\mathrm{t}=1}^{\mathrm{n}}(\log(\widehat{\mathrm{RV}}_\mathrm{t}) + \mathrm{RV_t}/\widehat{\mathrm{RV}}_\mathrm{t}) \qquad (16),$$

where $\mathrm{RV_t}$ and $\widehat{\mathrm{RV}}_\mathrm{t}$ denote actual and forecast volatility, respectively.

Model Confidence Set

While the above forecast measures provide a value that can be compared across different models, it is also important to consider any significant differences in the values. Hansen and Lunde (2005) propose the superior predictive ability (SPA) test to compare model accuracy. However, this approach requires the selection of a benchmark model, which can affect the result comparison. To address this shortcoming, Hansen et al. (2011) introduce the Model Confidence Set (MCS). The MCS removes the worst model sequentially according to rejection of the null hypothesis of equal predictive ability (EPA).

More specifically, the process of the MCS is given as. First, assume there are m_0 alternative forecasting models to be tested, so $M_0 = \{1, 2, \cdots, m_0\}$. Let $d_{ij,t}$ demote the loss function difference between any two models at time t:

$$d_{ij,t} = l_{i,t} - l_{j,t} \quad (i, j \in M_0) \qquad (17).$$

Second, the null hypothesis is set as any two models have EPA:

$$H_{0,M}: E\big(d_{ij,t} = 0\big), \; for\ all\ \ i, j \in M_0 \qquad (18),$$

$$H_{A,M}: E\left(d_{ij,t} \neq 0\right), \;\; for\ some\ \ i,j\ \in M_0 \qquad (19).$$

Third, in each step of MCS test, if the null hypothesis of equal predictive ability (EPA) is rejected at a given significant level, the worst forecasting model is removed sequentially until the null hypothesis of EPA is not rejected. One drawback of the test is that the predictive ability of any two forecast models is recalculated at each step of the process. To overcome this, Hansen et al. (2011) construct the Range and Semi-Quadratic statistics to test the above hypotheses:

$$T_R = \max_{i,j\, \in M_0} \left| \frac{\bar{d}_{i,j}}{\sqrt{\widehat{var}(\bar{d}_{i,j})}} \right| \;\; and \;\; T_{SQ} = \sum_{i,j\, \in M_0} \frac{\left(\bar{d}_{i,j}\right)^2}{\widehat{var}(\bar{d}_{i,j})} \qquad (20),$$

where $\bar{d}_{i,j}$ is the mean value of the loss functions difference, calculated as $\bar{d}_{i,j} = \frac{1}{M}\sum d_{ij,t}$.

4. Data and Empirical Results.

Data

We obtain the RV data from the Oxford-Man Institute of Quantitative Finance. We employ 5-minute RV data following Liu et al. (2015) for the stock indices of the UK (FTSE), Japan (N225), the US (SPX), Germany (DAX), China (SSEC), India (NSEI), Brazil (BSVP) and Mexico (MXX). We use the RV data in logarithmic form, log-RV, which produces a more normal distribution. The data is obtained over the sample period from 1st November 2006 to 31st October 2020. For the initial in-sample period, we use 1st November 2006 to 31st October 2010, while the out-of-sample forecast period covers 1st November 2010 to 31st October 2020.

Table 1 presents the summary statistics of log-RV for each index. All series exhibit a non-normal distribution with excess kurtosis, are right-skewed and present a significant Jarque-Bera test statistic at the 1% level. The first-order autocorrelation statistic indicates a reasonably high degree of persistence. Figure 1 provides the log-RV time-series plots, with noticeable increases in 2008 and 2020 as the financial crisis and global covid-19 pandemic unfold.

5. Empirical Results.

5.1. In-sample results

We begin with estimates of RV for the eight indexes using HAR and HAR-free models with a fixed lag structure model estimated by OLS, as well as flexible AR(22) and AR(100) models selected using the Lasso, adaptive Lasso, group Lasso and ordered Lasso approaches. As noted above, the tuning parameter, λ, is estimated using the minimum error of the CV. As this generates a substantial number of coefficients, we represent this with coefficient plots.

The estimated coefficients are plotted as line graphs in Figure 2 (a) and (b), where the coefficients are divided into three figures according to the model type for each index. The upper figure represents the fixed coefficients of the standard HAR and HAR-free models. The middle figure presents the flexible coefficients of AR(22) using Lasso, adaptive Lasso, group Lasso and ordered Lasso approaches. The lower figure is the AR(100), again, estimated using the alternative Lasso approaches. For the fixed coefficients across all indexes, the value of the first six coefficients of the HAR model and HAR-free model decrease with the lag, while the coefficients beyond lag seven are close to zero. For the flexible AR(22) structure, all four Lasso-based methods select coefficients from lag 1 to 5 with a declining trend. The Lasso and adaptive Lasso have a similar pattern with several longer lagged coefficients selected, although many are set to zero. For the grouped Lasso, the selected groups are {1}, {2-5}, {6-22}, which follow the HAR model. The coefficients of the ordered Lasso are monotonically decreasing with lag length. The lower figure, for the AR(100), reveals that lags beyond lag 22 are rarely selected across the Lasso methods. For example, for the grouped ordered Lasso models, the coefficients are close or exactly equal to zero. However, the Lasso and adaptive Lasso models do select some longer lags, with the adaptive Lasso selecting more than the Lasso.

Table 2 provides the in-sample MSE and QLIKE loss functions. Here, the standard HAR model is set as the benchmark model against which to compare the loss function values.

Thus, a value below one indicates preference for the alternative model, while above one indicates preference for the HAR model. According to both the MSE and QLIKE, the AR(100) using the adaptive Lasso provides the smallest in-sample error across all indexes, while the AR(22) with the Lasso method also provides a good performance. This indicates that some predictive information exists beyond lag 22 across the RV series.

The in-sample results suggest two key features. First, the flexible AR models show that while the first 22 lags are more frequently selected than longer lags, there are some longer lags that provide relevant forecasting information. Second, the flexible lag models slightly improve fit over fixed lag models. This result is consistent with the work of Audrino and Knaus (2016) and Audrino et al. (2019).

5.2. Out-of-sample results

The forecasts are conducted over the out-of-sample period from 1st November 2010 to 31st October 2020 using both a rolling (fixed window) and recursive (expanding window) approach. The initial window size is 500 daily observations. Moreover, while emphasis is typically on daily volatility forecasts, we also consider both weekly and monthly forecasts. In generating these multi step-ahead forecasts, we replace RV_{t+1} on the left-head side of equation (8) with $RV_{t+h}^{h} = \frac{1}{h}\sum_{i=1}^{h} RV_{t-h+i}$, where the forecast horizon, $h = 1, 5\ and\ 22$. The forecasting performance is measured by the MSE and QLIKE loss function where the standard HAR model is regarded as a benchmark, while the MCS test selects optimal models with EPA.

Tables 3 and 4 present the MSE values using the rolling and recursive window approaches respectively for the daily, weekly and monthly horizon (h=1, 5, 22). Tables 3 and 4 present broadly consistent results. For one-day-ahead forecasts, the AR(100) ordered Lasso performs the best across almost all series. The Lasso and adaptive Lasso for both AR(22) and AR(100) perform poorly. The two fixed lag structure models, HAR model and HAR-free model, exhibit similar forecasting performance. For the one-week-ahead forecasts, the Lasso-methods overwhelmingly outperform the HAR and HAR-free models, especially, the ordered Lasso for the AR(22), which dominates across all stock indexes. For the one-month-ahead forecast, again, the HAR model and HAR-free model perform poorly and the ordered Lasso performs best. Tables 5 and 6 present the equivalent results for the QLIKE measure and show similar results to those in Tables 3 and 4. There is predominately a preference for the ordered Lasso model with AR(100) at the daily horizon and AR(22) at the weekly and monthly horizons.

The MCS test results for the rolling window approach are presented in Tables 7 and 8 for the MSE and QLIKE metrics, respectively. In the tables, the value of 1 means the optimal model is chosen, while the MCS test also chooses a subset of models with EPA at the 75% confidence level. Generally, the Lasso-based models perform significantly better than the HAR model. Specifically, the AR(100) ordered Lasso method performs the best at the daily horizon. For weekly forecasting, evidence supports the superiority of the ordered Lasso AR(22). At the monthly horizon, the ordered Lasso AR(22) again performs the best, although given the greater smoothness of the monthly volatility, several models have EPA for the monthly forecast. Notwithstanding this, the Lasso models are preferred over the HAR and HAR free models.

Tables 9 and 10 present the MCS test for the recursive (increasing window) approach. In comparison with Tables 7 and 8, similar results are obtained. Again, the AR(100) using ordered Lasso is preferred at the daily horizon. For the weekly results, the ordered Lasso AR(22) is generally preferred, although the AR(100) ordered Lasso performs well for the NSEI. The same models are also preferred at the monthly horizon, where the Lasso models dominate the standard HAR approaches.

In sum, across the different forecast metrics, the ordered Lasso AR(100) model is preferred at the daily level, while at the weekly and monthly horizons, the ordered Lasso AR(22) dominates. It is also of interest to note that there are no clear differences between the four develop and four emerging markets in terms of the forecast results.

6. Risk Management Application.

To further assess the forecast performance of the Lasso-based models in comparison to the HAR model, we consider a risk management perspective and the Value-at-Risk (VaR) measure. VaR is designed to measure and monitor risk by considering the potential loss occurring with a given possibility over a specific time frame. The VaR of an asset is calculated as:

$$VaR = \mu_t + \sigma_t N(\alpha) \quad (21),$$

where μ_t is the mean of the log-return, σ_t is the forecast volatility, and $N(\alpha)$ defines the left αth quantile of the normal distribution.

To evaluate the accuracy of VaR forecasts, we utilise three tests. First, we compute the failure rate for daily returns, which is the number of times daily returns exceed the forecasted VaR. Second, we compute the Dynamic Quantile (DQ) test of Engle and Manganelli (2004) to examine whether VaR violations are correlated. The hit sequence is defined as follow:

$$Hit_t = I(r_t < -VaR_t) - a \quad (21),$$

where the value $(1 - a)$ is when actual returns are less than the VaR quantile, with the value $(-a)$ otherwise. The expected value of Hit_t is zero, while the sequence should be uncorrelated. Thus, an AR model for the Hit_t sequence is estimated, where the parameters are expected to be zero. The DQ test statistic is calculated as:

$$DQ = \frac{\hat{\beta}' X' X \hat{\beta}}{a(1-a)} \sim \chi^2(k) \quad (22),$$

where X are the explanation variables and $\hat{\beta}$ the OLS estimates.[1] The DQ test follows χ^2 distribution with degree of freedom equal to the number of parameters.

[1] The *X* variables includes the lags of the Hit_t as well as lags of other relevant variables such as returns and squared returns.

Third, we compute the Weibull test of Christoffersen and Pelletier (2004). The main idea of this duration-based test is that the duration between VaR violations which should be independent and not cluster. Thus, VaR violations should be memoryless and should follow an exponential distribution. Christoffersen and Pelletier (2004) consider a Weibull distribution, which has the density function:

$$f_w(x, a, b) = a^b b x^{b-1} e^{-(ax)^b} \qquad (23),$$

where the exponential distribution is a special case when $b = 1$. The null hypothesis of VaR violations being independent and memoryless corresponds to $b = 1$.

The daily VaR forecasts for the rolling window are presented in Table 11 for the 1% and 5% levels. In regard of the lowest average failure rate, the AR(100) ordered Lasso model is preferred while the ordered and grouped Lasso models for both AR lags tend to perform better than alternative Lasso and HAR models. In term of the Weibull test and DQ test, all models fail the DQ test across all series, while equally, no model passed the Weibull test for all series, although both the HAR and HAR-free model performs better and only indicate significance for the SSEC and N225. For the weekly VaR forecasts, the HAR and HAR-free models perform poorly, having the highest average failure rate and having four markets significant at 1% level and all market significant at 5% on the Weibull test. The Lasso models improve accuracy of the weekly VaR, compared with HAR model. Notably, the AR(22) ordered Lasso model performs the best at the 5% VaR level. Examining monthly VaR results, as noted above, the smoothness of monthly volatility results in no VaR exceedances. Given this, all models do not reject the null hypothesis of VaR violations autocorrelation (DQ test) and independence and memoryless (Weibull test).

Table 12 presents the VaR results for the recursive approach, with the results generally similar to those reported above. For the daily VaR forecasts, the AR(100) with the ordered Lasso method achieve the lowest average failure rate. The HAR model and HAR-free model show only one market significant on the Weibull test at 1% VaR level. Examining the weekly VaR forecasts, the HAR model and HAR-free model perform worse in term of the average failure rate and Weibull test. For the monthly VaR results, identically with Table 11, no returns exceed the VaR forecasts, no models reject the null hypothesis of the Weibull test and DQ test.

7. Summary and Conclusion.

The Lasso approach, originally developed in the field of computational statistics, is increasingly applied to financial time-series data. This approach has the potential to improve model selection and thereby improved forecasts. This paper, therefore, considers whether the Lasso-based method can indeed improve volatility forecasts over the HAR model that has come dominate the literature. Specifically, we compare the forecasting ability between the flexible lag models using four variants of the Lasso approach and fixed lag models using two HAR models for realised volatility series across eight international markets. This paper thus, extends the work of Audrino and Knaus (2016) and Croux et al. (2018).

The in-sample results support an AR(100) model estimated using the adaptive Lasso approach. However, all the flexible lags models provide a slightly improved fit over the HAR models using two loss functions. The coefficients plots suggest that while the first 22 lags are the most important, the Lasso AR(100) models indicate that some longer lags do contain relevant information. The out-of-sample results present a relatively consistent pattern across the MSE, QLIKE, MCS and VaR measures. The AR(100) ordered Lasso is preferred for daily forecasts, while the AR(22) ordered Lasso is preferred for the weekly and monthly forecasts.

The current literature suggests that the HAR model is the preferred approach for modelling and forecasting volatility. The HAR model is based on a fixed lag approach, with lagged volatility modelled on a day, week and month basis. This model ignores the possibility that longer lags may contain information and that the restricted lag structure may not be the most appropriate. The results presented here suggest that a more flexible lag approach does improve forecast performance over the HAR model. As our results further show, this has implications for risk management.

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Figure 1: The plots of the time series of log-RV of eight market index from 1st November 2006 to 31st October

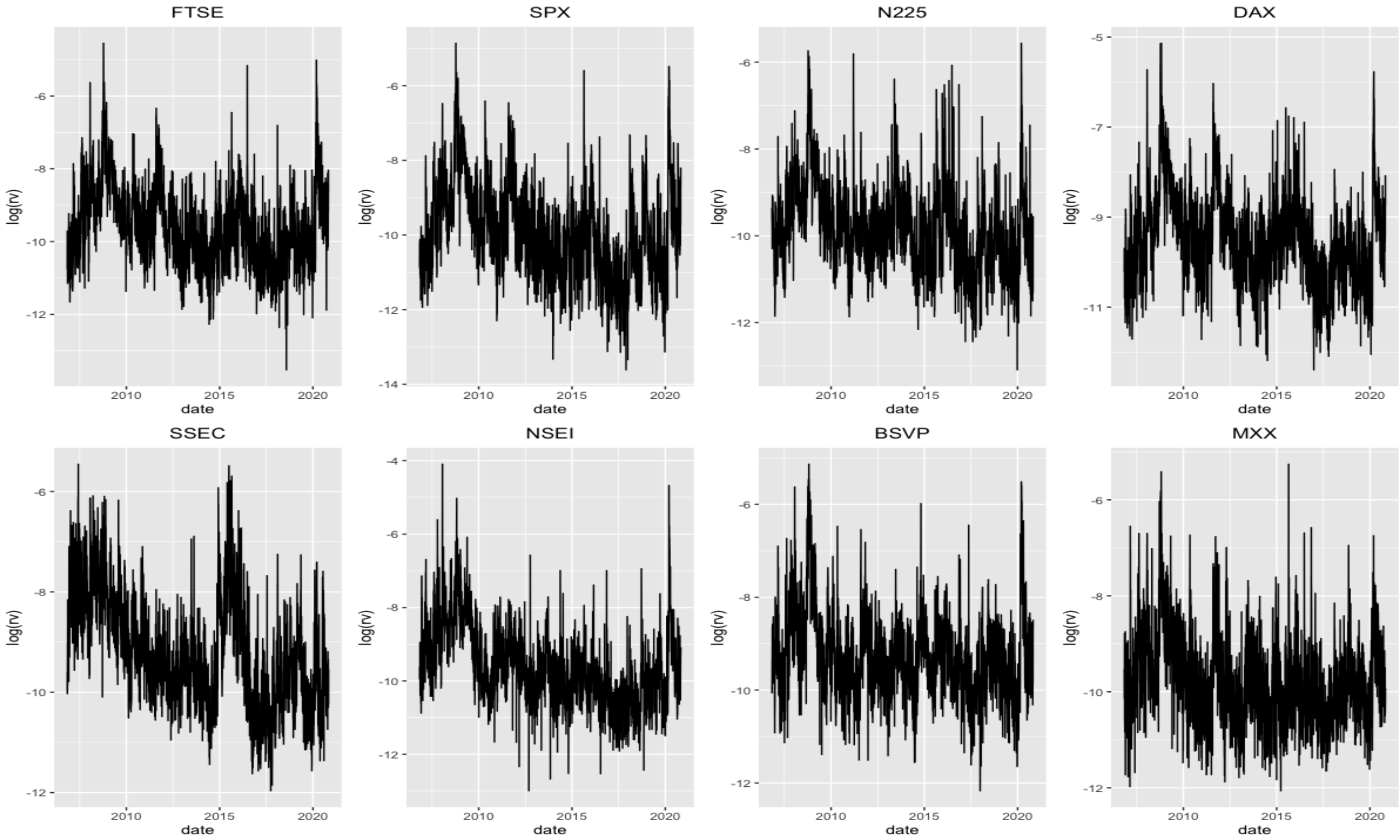

Figure 2 (a): The plots of coefficients for eight stock index in the in-sample period

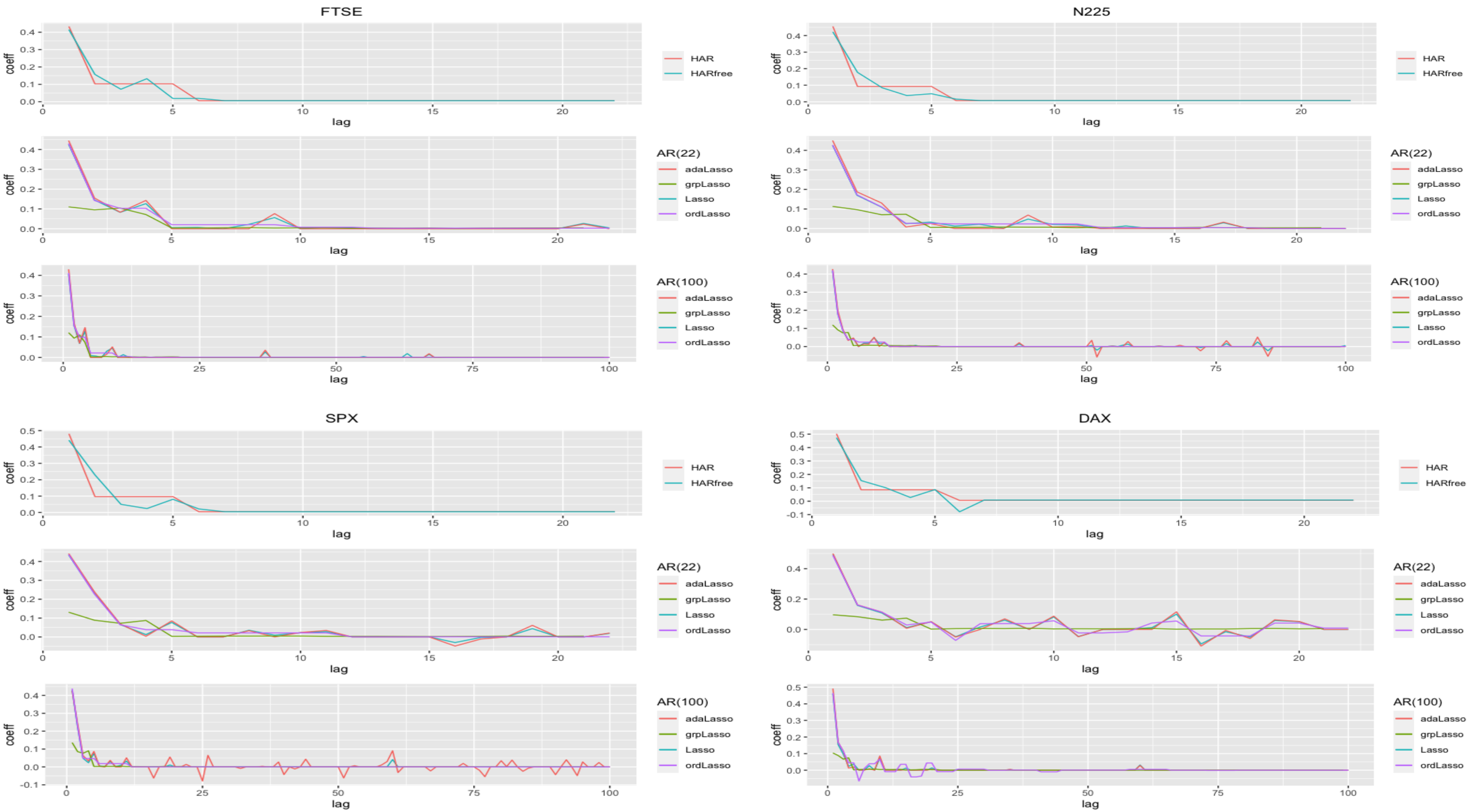

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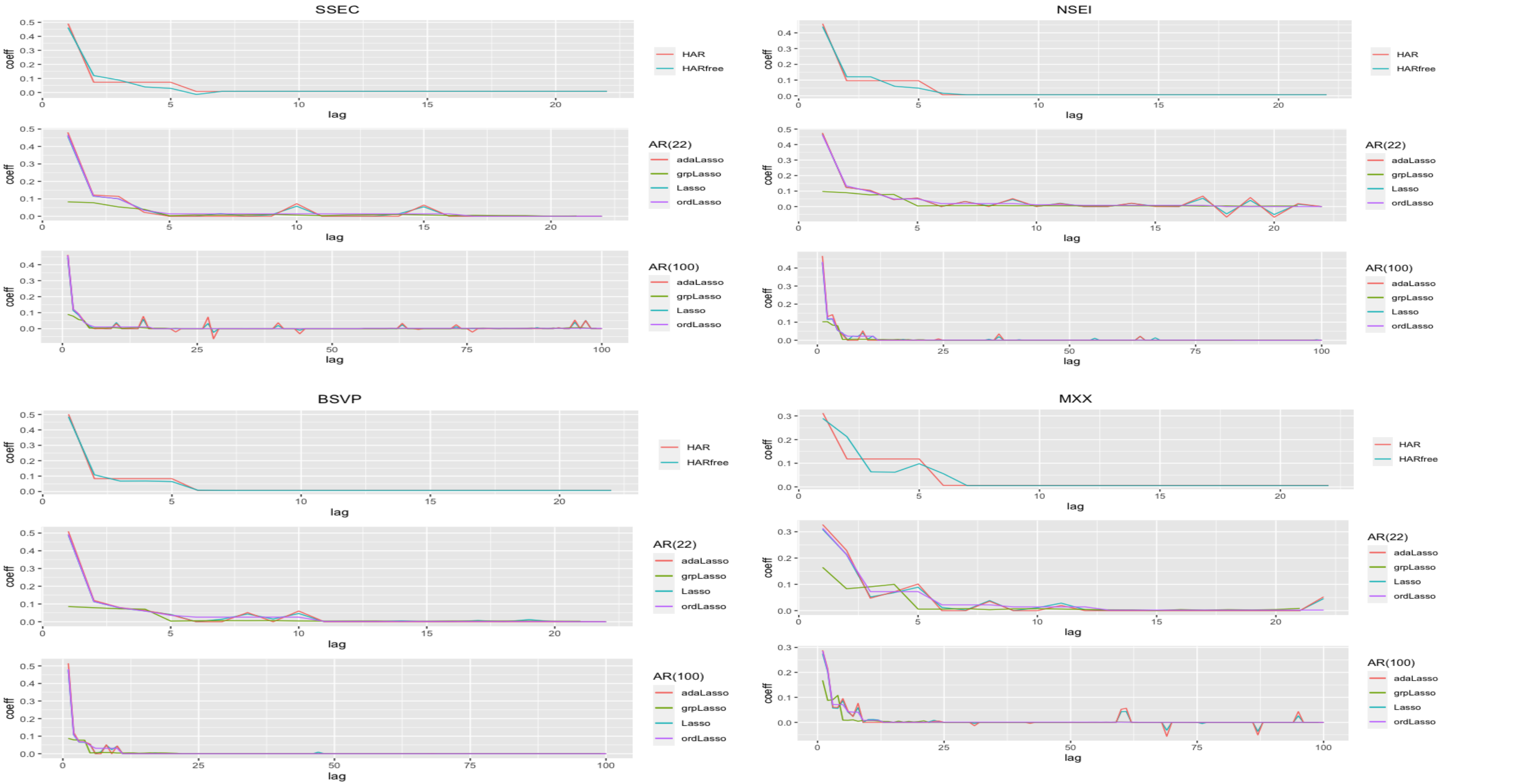
Figure 2 (b): The plots of coefficients for eight stock index in the in-sample period



Note: This figure shows the estimated coefficients for eight stock index in the in-sample period from 1st November 2006 to 31st October 2010. The upper figure is the fixed coefficients of the standard HAR model and HAR-free model. The middle figure is the flexible coefficients of AR (22) using Lasso-based method, and the bottom figure is the AR (100) estimated by Lasso-based method. AdaLasso means the adaptive Lasso method, grpLasso means the grouped Lasso method, ordLasso means the ordered Lasso method, respectively.

Table 1: Statistics Description of Log-RV

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
Mean	-9.6253	-9.9753	-9.8512	-9.4991	-9.2238	-9.6553	-9.2901	-9.8426
Std.Dev.	1.0289	1.2418	0.9759	0.9824	1.0784	1.0239	0.8881	0.8965
Kurtosis	0.7981	0.3428	0.9749	0.6954	-0.1832	0.9356	1.8284	1.0329
Skewness	0.6497	0.4930	0.6467	0.4802	0.4387	0.7223	0.8595	0.7881
Median	-9.7546	-10.0879	-9.9397	-9.5673	-9.3426	-9.8042	-9.3765	-9.9778
25%-quantile	-10.355	-10.841	-10.506	-10.159	-9.986	-10.359	-9.849	-10.490
75%-quantile	-9.021	-9.216	-9.319	-8.910	-8.438	-9.063	-8.836	-9.316
AutoCorr$_{lag=1}$	0.7600	0.8324	0.7748	0.8021	0.8383	0.8269	0.7671	0.6407
Jarque-Bera	343.41***	159.93***	374.53***	208.22***	113.91***	106.09***	906.68***	520.49***
Obs.	3537	3515	3419	3545	3402	3465	3448	3511

Note: This table repots the summary statistics of log-RV of eight different stock index for the whole period from 1st November 2006 to 31st October 2020. *** indicate significant level at 1%.

Table 2: In-sample estimation error of MSE and QLIKE

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				MSE				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9915	0.9821	0.9902	0.9864	0.9943	0.9946	0.9990	0.9862
AR(22)-Lasso	1.0169	0.9908	1.0041	0.9798	1.0310	0.9808	0.9919	1.0038
AR(22)-adalasso	1.0183	0.9884	1.0086	0.9798	1.0315	0.9803	0.9942	1.0046
AR(22)-grpLasso	1.0339	1.0236	1.0238	1.0487	1.0460	0.9931	1.0084	1.0159
AR(22)-ordLasso	1.0216	1.0022	1.0063	0.9987	1.0372	0.9789	0.9954	1.0096
AR(100)-Lasso	**0.9765***	0.9687	0.9647	0.9706	0.9473	0.9788	0.9916	0.9516
AR(100)-adalasso	0.9788	**0.8920***	**0.9504***	**0.9499***	**0.9287***	**0.9721***	0.9934	**0.9439***
AR(100)-grpLasso	0.9988	0.9987	1.0005	1.0018	0.9860	1.0007	1.0063	0.9901
AR(100)-ordLasso	0.9861	0.9769	0.9826	0.9500	0.9847	0.9963	**0.9915***	0.9806
				QLIKE				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9886	0.9819	0.9894	0.9864	0.9938	0.9936	0.9988	0.9856
AR(22)-Lasso	0.9874	0.9675	0.9845	0.9534	1.0327	0.9728	**0.9888***	0.9947
AR(22)-adalasso	0.9883	0.9653	0.9879	0.9529	1.0333	0.9715	0.9905	0.9945
AR(22)-grpLasso	1.0070	1.0015	1.0082	1.0226	1.0478	0.9847	1.0080	1.0076
AR(22)-ordLasso	0.9927	0.9774	0.9857	0.9745	1.0400	**0.9698***	0.9911	0.9989
AR(100)-Lasso	**0.9740***	0.9730	0.9653	0.9724	0.9461	0.9814	0.9954	0.9544
AR(100)-adalasso	0.9746	**0.8932***	**0.9507***	**0.9485***	**0.9272***	0.9729	0.9946	**0.9459***
AR(100)-grpLasso	0.9993	1.0028	1.0032	1.0047	0.9850	1.0015	1.0107	0.9912
AR(100)-ordLasso	0.9847	0.9780	0.9804	0.9506	0.9846	0.9962	0.9927	0.9804

Note: This table reports the MSE and QLIKE value of eight RV index for all forecasting models considered for the in-sample period from 1st November 2006 to 31st October 2010. The standard HAR model is regarded as benchmark against other forecasting models, the forecasting model with best performance is highlighted with *.

Table 3: Out-of-sample forecasting evaluation using MSE of rolling window forecasting

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				h=1				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	1.0011	1.0016	1.0020	1.0072	1.0041	0.9945	1.0088	0.9958
AR(22)-Lasso	1.2796	1.0136	1.0664	1.1650	1.1088	1.2088	1.0920	1.3665
AR(22)-adaLasso	1.2562	0.9917	0.9960	1.1235	0.7035	1.1500	1.0254	1.3578
AR(22)-grpLasso	0.7748	**0.5588***	0.6312	0.6484	0.6764	0.7803	0.6104	0.9194
AR(22)-ordLasso	0.9450	0.5757	0.7266	0.9811	0.7118	1.1335	0.7213	0.9949
AR(100)-Lasso	1.2641	0.9976	1.0440	1.1452	1.1056	1.2016	1.0601	1.3612
AR(100)-adalasso	1.2322	0.7876	1.0239	1.0438	0.9155	1.1694	**0.5852***	1.3484
AR(100)-grpLasso	0.7572	**0.5588***	0.6002	0.6527	0.6528	0.7379	0.6581	**0.8905***
AR(100)-ordLasso	**0.6335***	0.6574	**0.5240***	**0.6301***	**0.6420***	**0.7215***	1.0280	0.9397
				h=5				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9874	0.9733	0.9869	0.9747	0.9909	0.9834	0.9916	0.9733
AR(22)-Lasso	0.4391	0.6759	0.3720	0.4066	0.3896	0.4151	**0.3691***	0.7276
AR(22)-adalasso	0.4267	0.7340	0.3660	0.4037	0.3829	0.4105	0.4752	0.7340
AR(22)-grpLasso	0.4342	0.5000	0.4172	0.6040	0.3847	0.3859	0.6716	0.5000
AR(22)-ordLasso	**0.4251***	**0.4274***	**0.3366***	**0.3816***	**0.3805***	**0.3431***	0.4082	**0.4584***
AR(100)-Lasso	0.4396	0.4364	0.3678	0.4045	0.3966	0.3686	0.3870	0.4623
AR(100)-adalasso	0.5559	0.6719	0.3907	0.4950	0.3933	0.4778	0.5384	0.6719
AR(100)-grpLasso	0.5269	0.5453	0.4515	0.6683	0.4196	0.4091	0.8376	0.5453
AR(100)-ordLasso	0.5036	0.4626	0.7203	1.3994	0.7680	0.3811	0.9855	0.4626
				h=22				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9929	1.0007	0.9923	0.9994	0.9934	0.9970	0.9969	0.9966
AR(22)-Lasso	0.6150	0.5879	0.5803	0.5406	0.7171	**0.5420***	0.7696	0.5792
AR(22)-adalasso	0.6851	0.5898	0.6365	0.5403	0.8478	0.8091	0.7943	0.6275
AR(22)-grpLasso	0.6974	0.6654	0.6124	0.5797	0.7514	0.5493	0.9393	0.5935
AR(22)-ordLasso	**0.6018***	**0.5875***	**0.5678***	**0.5303***	0.7346	0.5537	**0.7509***	**0.5556***
AR(100)-Lasso	0.6584	0.5944	0.5942	0.5364	**0.7083***	0.5670	0.7862	0.6193
AR(100)-adalasso	0.8292	0.6335	0.6186	0.6062	0.7117	0.7718	0.9050	0.7243
AR(100)-grpLasso	0.7161	0.6785	0.6254	0.5414	0.7432	0.5733	1.0498	0.6155
AR(100)-ordLasso	0.7094	0.8017	0.9535	0.5341	0.7697	0.5785	0.8252	0.5970

Note: This table reports the forecasting evaluation (MSE) of eight RV index for all forecasting models considered using rolling window approach (window size = 1000) over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st November 2010 to 31st October 2020. The standard HAR model is regarded as benchmark and the forecasting model with the best performance is highlighted with *.

Table 4: Out-of-sample forecasting evaluation using MSE of increasing window forecasting

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				h=1				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9964	0.9940	1.0001	0.9999	0.9927	0.9946	1.0017	0.9932
AR(22)-Lasso	1.2830	1.0081	1.0713	1.1623	1.1270	1.1858	1.0917	1.3671
AR(22)-adalasso	1.2418	0.9793	0.9872	1.1074	0.6424	1.1240	1.0223	1.3496
AR(22)-grpLasso	0.7250	0.5354	0.5921	0.6017	0.6898	0.7561	**0.5708***	0.9346
AR(22)-ordLasso	1.1679	0.5885	0.9578	1.1026	0.6687	1.1559	0.9744	1.2627
AR(100)-Lasso	1.2691	0.9953	1.0496	1.1477	1.1238	1.1800	1.0678	1.3635
AR(100)-adalasso	1.2404	0.7267	1.0135	1.0227	0.9026	1.1511	0.4372	1.3516
AR(100)-grpLasso	0.7064	0.5321	0.5614	**0.5986***	0.6468	0.7141	0.5764	0.8987
AR(100)-ordLasso	**0.5581***	**0.4169***	**0.4783***	0.9694	**0.4650***	**0.4548***	0.8090	**0.6305***
				h=5				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9834	0.9671	0.9847	0.9710	0.9837	0.9810	0.9833	0.9671
AR(22)-Lasso	0.3597	0.3899	0.3505	0.3528	0.3376	0.3487	0.3417	0.3899
AR(22)-adalasso	0.3970	0.6842	0.3559	0.3531	0.3886	0.3921	0.4361	0.6842
AR(22)-grpLasso	0.4076	0.4366	0.3912	0.4412	0.3610	0.3461	0.4930	0.4366
AR(22)-ordLasso	**0.3275***	**0.3754***	**0.3346***	**0.3296***	**0.3019***	**0.3382***	**0.3255***	**0.3754***
AR(100)-Lasso	0.3663	0.3970	0.3481	0.3575	0.3396	0.3546	0.3422	0.3970
AR(100)-adalasso	0.5111	0.6198	0.3483	0.4387	0.3398	0.4453	0.4644	0.6198
AR(100)-grpLasso	0.4325	0.4636	0.4118	0.4860	0.3900	0.3603	0.5811	0.4636
AR(100)-ordLasso	0.3371	0.3857	0.3485	0.4848	0.3542	0.3406	0.3972	0.3857
				h=22				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9893	0.9932	0.9872	0.9971	0.9887	0.9945	0.9876	0.9913
AR(22)-Lasso	0.5775	0.5478	0.5748	0.5135	0.6557	0.5205	0.6941	0.5591
AR(22)-adalasso	0.6331	0.5505	0.6253	0.5151	0.8312	0.7618	0.7126	0.5989
AR(22)-grpLasso	0.5957	0.5722	0.5887	0.5261	0.7013	**0.5130***	0.7462	0.5683
AR(22)-ordLasso	**0.5717***	0.5439	**0.5709***	**0.5101***	0.6712	0.5139	**0.6881***	**0.5496***
AR(100)-Lasso	0.5880	**0.5359***	0.5766	0.5114	**0.6402***	0.5331	0.6898	0.5689
AR(100)-adalasso	0.7406	0.5746	0.5960	0.5807	0.6602	0.7163	0.7886	0.6689
AR(100)-grpLasso	0.6156	0.5776	0.5994	0.5191	0.6885	0.5227	0.7824	0.5802
AR(100)-ordLasso	0.5868	0.5433	0.6281	0.5111	0.6748	0.5345	0.6956	0.5598

Note: This table reports the forecasting evaluation (MSE) of eight RV index for all forecasting models considered using increasing window approach over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period 1st November 2010 to 31st October 2020. The standard HAR model is regarded as benchmark and the forecasting model with the best performance is highlighted with *.

Table 5: Out-of-sample forecasting evaluation using QLIKE of rolling window forecasting

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				h=1				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	1.0009	1.0018	1.0010	1.0075	1.0029	0.9950	1.0086	0.9968
AR(22)-Lasso	1.2850	1.0339	1.0719	1.1574	1.1197	1.2198	1.0907	1.3997
AR(22)-adaLasso	1.2589	1.0092	0.9939	1.1144	0.7022	1.1563	1.0207	1.3895
AR(22)-grpLasso	0.7711	0.5697	0.6338	0.6635	0.6780	0.7774	0.6129	0.9292
AR(22)-ordLasso	0.9395	**0.4586***	0.7278	0.6585	**0.5707***	1.1405	0.7167	1.0087
AR(100)-Lasso	1.2663	1.0131	1.0484	1.1344	1.1200	1.2215	1.0571	1.3939
AR(100)-adalasso	1.2307	0.7975	1.0269	1.0321	0.9141	1.1853	**0.6094***	1.3790
AR(100)-grpLasso	0.7520	0.5686	**0.6054***	0.6543	0.6440	0.7402	0.6644	**0.8996***
AR(100)-ordLasso	**0.7436***	0.9365	0.7927	**0.6538***	0.9711	**0.7253***	1.3687	1.0901
				h=5				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9880	0.9753	0.9862	0.9750	0.9904	0.9860	0.9903	0.9753
AR(22)-Lasso	0.4624	0.4241	0.3683	0.4116	0.4044	0.3759	**0.3689***	0.5185
AR(22)-adalasso	0.4438	0.7362	0.3753	0.4074	0.3895	0.4172	0.4791	0.7362
AR(22)-grpLasso	0.5152	0.5109	0.4358	0.6358	0.3899	0.3920	0.6924	0.5109
AR(22)-ordLasso	**0.3968***	**0.4121***	**0.3441***	**0.3926***	**0.3858***	**0.3575***	0.4929	**0.4970***
AR(100)-Lasso	0.3991	0.4439	0.3778	0.4101	0.4025	0.3864	0.3877	0.5100
AR(100)-adalasso	0.5646	0.6765	0.3963	0.5029	0.3907	0.4881	0.5466	0.6765
AR(100)-grpLasso	0.5484	0.5570	0.4735	0.7025	0.4266	0.4165	0.8615	0.5570
AR(100)-ordLasso	0.4094	0.4707	0.7335	1.4427	0.7768	0.3877	1.0171	0.4990
				h=22				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9916	0.9997	0.9910	0.9988	0.9948	0.9958	0.9969	0.9963
AR(22)-Lasso	0.6233	0.6072	0.5816	0.5378	0.8115	**0.5368***	0.7720	0.5919
AR(22)-adalasso	0.7024	0.6124	0.6390	0.5379	0.7912	0.7800	0.7986	0.6432
AR(22)-grpLasso	0.7251	0.6928	0.6205	0.5896	0.7664	0.5379	0.9498	0.6092
AR(22)-ordLasso	**0.6104***	**0.6056***	**0.5691***	**0.5282***	0.7521	0.5439	**0.7543***	**0.5687***
AR(100)-Lasso	0.6655	0.6086	0.5985	0.5423	**0.7246***	0.5590	0.7906	0.6322
AR(100)-adalasso	0.8470	0.6462	0.6217	0.6143	0.7323	0.7482	0.9152	0.7384
AR(100)-grpLasso	0.7468	0.7107	0.6368	0.5555	0.7630	0.5621	1.0657	0.6324
AR(100)-ordLasso	0.7255	0.8386	0.9678	0.5472	0.7913	0.5680	0.8416	0.6094

Note: This table reports the forecasting evaluation (QLIKE) of eight RV index for all forecasting models considered using rolling window approach (window size = 1000) over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from1st November 2010 to 31st October 2020. The standard HAR model is regarded as benchmark, the forecasting model with the best performance is highlighted with *.

Table 6: Out-of-sample forecasting evaluation using QLIKE of increasing window forecasting

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				h=1				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9967	0.9938	0.9999	1.0007	0.9919	0.9949	1.0014	0.9938
AR(22)-Lasso	1.2927	1.0244	1.0792	1.1549	1.1412	1.1987	1.0894	1.4018
AR(22)-adalasso	1.2483	0.9935	0.9881	1.0985	0.6472	1.1331	1.0181	1.3821
AR(22)-grpLasso	0.7229	0.5391	0.5936	0.5959	0.6925	0.7551	0.5682	0.9468
AR(22)-ordLasso	1.1729	0.5884	0.9631	1.0939	0.6710	1.1645	0.9707	1.2910
AR(100)-Lasso	1.2761	1.0086	1.0570	1.1385	1.1416	1.2017	1.0657	1.3985
AR(100)-adalasso	1.2451	0.7313	1.0200	1.0113	0.9033	1.1693	**0.4435***	1.3852
AR(100)-grpLasso	0.7024	0.5337	0.5640	**0.5919***	0.6511	0.7182	0.5758	0.9099
AR(100)-ordLasso	**0.5595***	**0.4235***	**0.4821***	0.9570	**0.4727***	**0.4738***	0.8085	**0.6387***
				h=5				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9841	0.9694	0.9851	0.9710	0.9833	0.9837	0.9823	0.9694
AR(22)-Lasso	0.3660	0.3935	0.3593	0.3529	0.3441	0.3650	0.3365	0.3935
AR(22)-adalasso	0.4069	0.6843	0.3618	0.3529	0.3954	0.4014	0.4304	0.6843
AR(22)-grpLasso	0.4141	0.4407	0.4013	0.4458	0.3655	0.3594	0.4980	0.4407
AR(22)-ordLasso	**0.3313***	**0.3785***	**0.3424***	**0.3293***	**0.3065***	0.3531	**0.3214***	**0.3785***
AR(100)-Lasso	0.3724	0.4011	0.3584	0.3577	0.3468	0.3740	0.3373	0.4011
AR(100)-adalasso	0.5136	0.6198	0.3577	0.4380	0.3446	0.4612	0.4619	0.6198
AR(100)-grpLasso	0.4391	0.4681	0.4233	0.4908	0.3963	0.3764	0.5887	0.4681
AR(100)-ordLasso	0.3406	0.3886	0.3572	0.4883	0.3595	**0.3376***	0.3952	0.3886
				h=22				
HAR model	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
HAR-free model	0.9888	0.9921	0.9861	0.9966	0.9891	0.9936	0.9871	0.9912
AR(22)-Lasso	0.5707	0.5463	0.5757	0.5081	0.6722	0.5208	0.6770	0.5677
AR(22)-adalasso	0.6328	0.5500	0.6251	0.5103	0.8554	0.7445	0.6964	0.6096
AR(22)-grpLasso	0.5936	0.5760	0.5903	0.5298	0.7208	**0.5112***	0.7354	0.5771
AR(22)-ordLasso	**0.5653***	**0.5440***	**0.5713***	**0.5046***	0.6898	0.5124	**0.6709***	**0.5589***
AR(100)-Lasso	0.5817	0.5528	0.5807	0.5130	**0.6631***	0.5337	0.6737	0.5773
AR(100)-adalasso	0.7371	0.5762	0.6002	0.5834	0.6862	0.7085	0.7723	0.6766
AR(100)-grpLasso	0.6178	0.5868	0.6040	0.5274	0.7108	0.5236	0.7783	0.5893
AR(100)-ordLasso	0.5821	0.5564	0.6336	0.5155	0.7027	0.5323	0.6808	0.5687

Note: This table reports the forecasting evaluation (QLIKE) of eight RV index for all forecasting models considered using increasing window approach over daily, weekly and monthly horizons (h=1, 5 and 22) and the out-of-sample period from 1st November 2010 to 31st October 2020. The standard HAR model is regarded as benchmark, the forecasting model with the best performance is highlighted with *.

Table 7: The Model Confidence Set test of MSE criterion for rolling window forecasting

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				h=1				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-ordLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**1.0000***	0.0000
AR(100)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.3382	0.0000	**1.0000***
AR(100)-ordLasso	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	0.0000	0.0000
				h=5				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	0.0000	0.0000	0.0000	0.7598	0.0000	0.0000	**1.0000***	0.0000
AR(22)-adalasso	0.0000	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.0000	0.0000
AR(22)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-ordLasso	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	0.0000	**1.0000***
AR(100)-Lasso	0.0000	0.0000	0.0000	0.0000	0.4338	0.0000	0.0000	0.0000
AR(100)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-ordLasso	0.0000	0.6980	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
				h=22				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	0.0000	**1.0000***	0.9266	0.8238	**1.0000***	0.3450	0.0898	**1.0000***
AR(22)-adalasso	0.0000	0.0830	0.0000	0.0000	0.0000	**1.0000***	0.0032	0.0000
AR(22)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5662
AR(22)-ordLasso	**1.0000***	**1.0000***	**1.0000***	**1.0000***	0.9064	0.9268	**1.0000***	**1.0000***
AR(100)-Lasso	0.0000	0.7380	0.3464	0.3496	0.8944	0.4378	**1.0000***	0.0000
AR(100)-adalasso	0.0000	0.0000	0.0000	0.0000	0.8080	0.0000	0.7376	0.0000
AR(100)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
AR(100)-ordLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.5780	0.0000	0.6440

Note: This table reports the MSC test in term of MSE criterion for eight RV index over daily, weekly and monthly horizons (h=1, 5 and 22). The forecasting models with EPA at 75% confidence level is highlighted in table.

Table 8: The Model Confidence Set test of MSE criterion for rolling window forecasting

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				h=1				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-ordLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**1.0000***	0.0000
AR(100)-grpLasso	0.0000	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.0000	0.0000
AR(100)-ordLasso	**1.0000***	**1.0000***	**1.0000***	0.0000	**1.0000***	**1.0000***	0.0000	**1.0000***
				h=5				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	0.0000	0.3110	0.4856	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-adalasso	0.0000	0.0000	0.5088	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-ordLasso	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***
AR(100)-Lasso	0.0000	0.3052	0.9526	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-adalasso	0.0000	0.0000	0.8978	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-ordLasso	0.0000	0.7502	0.7260	0.0000	0.0000	0.0000	0.0000	0.2932
				h=22				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	0.0030	0.5900	0.5952	0.9998	1.0000*	0.0000	0.4958	1.0000
AR(22)-adalasso	0.0000	0.5900	0.0000	0.0000	0.0000	0.3350	0.0000	0.0000
AR(22)-grpLasso	0.5826	0.4416	0.5896	0.0000	0.0000	0.0000	0.0000	0.3482
AR(22)-ordLasso	**1.0000***	**1.0000***	**1.0000***	**1.0000***	0.3866	0.0000	**1.0000***	1.0000*
AR(100)-Lasso	**1.0000***	**1.0000***	**1.0000***	0.9748	0.6676	0.0000	**1.0000***	0.6060
AR(100)-adalasso	0.0000	0.3232	0.0080	0.0000	0.0000	0.0000	0.9816	0.0000
AR(100)-grpLasso	0.4538	0.4218	0.6894	0.0000	0.0000	**1.0000***	0.0000	0.0000
AR(100)-ordLasso	0.5674	**1.0000***	0.0000	0.9340	0.0000	0.0000	0.0000	**1.0000***

Note: This table reports the MSC test in term of QLIKE criterion for eight RV index over daily, weekly and monthly horizons (h=1, 5 and 22). The forecasting models with EPA at 75% confidence level is highlighted in table.

Table 9: The Model Confidence Set test of QIIKE criterion for rolling window forecasting

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				h=1				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-grpLasso	0.0000	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.0000	0.0000
AR(22)-ordLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**1.0000***	0.0000
AR(100)-grpLasso	0.0000	0.0000	**1.0000***	0.9336	0.0000	0.3184	0.7396	**1.0000***
AR(100)-ordLasso	**1.0000***	**1.0000***	0.0000	0.8774	**1.0000***	**1.0000***	0.0000	0.0000
h=5	**MSE**	**MSE**	**MSE**	**MSE**	**MSE**	**MSE**	**MSE**	**MSE**
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	0.0000	0.0000	0.0000	0.3517	0.1411	0.9294	**1.0000***	0.0000
AR(22)-adalasso	0.0000	0.0000	0.0000	0.4916	0.4478	0.0000	0.0000	0.0000
AR(22)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-ordLasso	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	0.0000	**1.0000***
AR(100)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.3896	0.0000	0.0000
AR(100)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-ordLasso	0.0000	0.6224	0.0000	0.0000	0.0000	0.6110	0.0000	0.0000
h=22	**MSE**	**MSE**	**MSE**	**MSE**	**MSE**	**MSE**	**MSE**	**MSE**
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	0.0000	0.9258	0.1892	0.0000	**1.0000***	**1.0000***	0.0000	**1.0000***
AR(22)-adalasso	0.0000	0.0376	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.2777	**1.0000***	0.0000	0.4708
AR(22)-ordLasso	**1.0000***	**1.0000***	**1.0000***	**1.0000***	0.9992	0.6852	**1.0000***	**1.0000***
AR(100)-Lasso	0.0000	0.8444	0.2798	0.0000	**1.0000***	0.3158	0.0000	0.0000
AR(100)-adalasso	0.0000	0.0000	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.0000
AR(100)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.2950	**1.0000***	0.0000	0.0000
AR(100)-ordLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.4782	0.0000	0.7344

Note: This table reports the MSC test in term of MSE criterion for eight RV index over daily, weekly and monthly horizons (h=1, 5 and 22). The forecasting models with EPA at 75% confidence level is highlighted in table.

Table 10: The Model Confidence Set test of QLIKE criterion increasing window forecasting

	FTSE	SPX	N225	DAX	SSEC	NSEI	BSVP	MXX
				h=1				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-ordLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-Lasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-adalasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	**1.0000***	0.0000
AR(100)-grpLasso	0.0000	0.0000	0.0000	**1.0000***	0.0000	0.0000	0.0000	0.0000
AR(100)-ordLasso	**1.0000***	**1.0000***	**1.0000***	0.0000	**1.0000***	**1.0000***	0.0000	**1.0000***
				h=5				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	0.0000	0.0000	0.9834	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-adalasso	0.0000	0.0000	0.8608	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-ordLasso	**1.0000***	**1.0000***	**1.0000***	**1.0000***	**1.0000***	0.0000	**1.0000***	**1.0000***
AR(100)-Lasso	0.0000	0.0000	0.9796	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-adalasso	0.0000	0.0000	0.8396	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-grpLasso	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-ordLasso	0.0000	0.0000	0.8382	0.0000	0.0000	**1.0000***	0.0000	0.2696
				h=22				
HAR model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-free model	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
AR(22)-Lasso	0.0566	0.8322	0.5288	0.8646	**1.0000***	0.0000	0.6154	**1.0000***
AR(22)-adalasso	0.0000	0.3262	0.0000	0.0000	0.0000	0.5948	0.0000	0.0000
AR(22)-grpLasso	0.3408	0.0000	0.2606	0.0000	0.0000	0.0000	0.0000	0.3688
AR(22)-ordLasso	**1.0000***	**1.0000***	**1.0000***	**1.0000***	0.4752	0.0000	**1.0000***	**1.0000***
AR(100)-Lasso	0.9994	**1.0000***	**1.0000***	0.4628	0.9588	0.0000	**1.0000***	0.7608
AR(100)-adalasso	0.0000	0.0000	0.0006	0.0000	0.0000	0.0000	0.0000	0.0000
AR(100)-grpLasso	0.0000	0.0000	0.5640	0.0000	0.0000	**1.0000***	0.0000	0.0000
AR(100)-ordLasso	**1.0000***	0.7770	0.0000	0.0000	0.0000	0.0000	0.8940	**1.0000***

Note: This table reports the MSC test in term of QLIKE criterion for eight RV index over daily, weekly and monthly horizons (h=1, 5 and 22). The forecasting models with EPA at 75% confidence level is highlighted in table.

Table 11: Summary of 1% and 5% VaR failure rates of rolling window forecasting

	1%			5%		
	Ave. failure rate	Sig. Weibull test	Sig. DQ test	Ave. failure rate	Sig. Weibull test	Sig. DQ test
			h=1			
HAR model	0.0369	SSEC, N225	All	0.0833	SSEC, NSEI	All
HAR-free model	0.0375	SSEC, N225	All	0.0830	SSEC, NSEI	All
AR (22)-Lasso	0.0369	FTSE, SPX, N225, DAX, SSEC, NSEI	All	0.0861	All	All
AR (22)-Lasso	0.0360	FTSE, SPX, N225, DAX, SSEC, NSEI	All	0.0860	All	All
AR (22)-grpLasso	0.0274	FTSE, SSEC	All	0.0801	N225, DAX, SSEC, NSEI, BSVP	All
AR (22)-ordLasso	0.0299	FTSE, SPX, N225, DAX, SSEC, NSEI,	All	0.0790	FTSE, N225, DAX, SSEC, NSEI, BSVP	All
AR (100)-Lasso	0.0370	FTSE, SPX, N225, DAX, SSEC NSEI	All	0.0861	FTSE, N225, DAX, SSEC, NSEI, BSVP, MXX	All
AR (100)-adaLasso	0.0334	FTSE, N225, DAX, SSEC, NSEI	All	0.0834	FTSE, N225, SSEC, NSEI, MXX	All
AR (100)-grpLasso	0.0277	FTSE, SPX, N225, SSEC	All	0.0777	N225, DAX, SSEC, NSEI, MXX	All
AR (100)-ordLasso	0.0267	SSEC, NSEI, BSVP	All	0.0781	FTSE, DAX, SSEC, NSEI	All
			h=5			
HAR model	0.0016	FTSE, SPX, N225, NSEI	All	0.0051	FTSE, SPX, N225, DAX, NSEI, BSVP, MXX	All
HAR-free model	0.0013	FTSE, SPX, N225, NSEI	All	0.0050	All	All
AR (22)-Lasso	0.0002	N225	All	0.0010	FTSE, SPX, N225, SSEC, MXX	All
AR (22)-adaLasso	0.0001	None	All	0.0013	FTSE, SPX, MXX	All
AR (22)-grpLasso	0.0002	N225	All	0.0010	SPX, N225, SSEC, MXX	All
AR (22)-ordLasso	0.0001	N225	All	0.0008	FTSE, SPX, N225, SSEC, MXX	All
AR (100)-Lasso	0.0002	N225	All	0.0013	FTSE, SPX, N225, SSEC, MXX	All
AR (100)-adaLasso	0.0003	N225	All	0.0015	FTSE, SPX, N225, MXX	All
AR (100)-grpLasso	0.0002	N225	All	0.0011	SPX, N225, SSEC, MXX	All
AR (100)-ordLasso	0.0002	None	All	0.0012	FTSE, SPX, N225, SSEC, MXX	All
			h=22			
HAR model	0.0000	None	All	0.0000	None	All
HAR-free model	0.0000	None	All	0.0000	None	All
AR (22)-Lasso	0.0000	None	All	0.0000	None	All
AR (22)-adaLasso	0.0000	None	All	0.0000	None	All
AR (22)-grpLasso	0.0000	None	All	0.0000	None	All
AR (22)-ordLasso	0.0000	None	All	0.0000	None	All
AR (100)-Lasso	0.0000	None	All	0.0000	None	All
AR (100)-adaLasso	0.0000	None	All	0.0000	None	All
AR (100)-grpLasso	0.0000	None	All	0.0000	None	All
AR (100)-ordLasso	0.0000	None	All	0.0000	None	All

Notes: this table provides the VaR results of rolling window test at the 1% and 5% VaR level. The average failure rate for each model over each index. The series are significant in the Weibull test and DQ test are listed.

Table 12: Summary of 1% and 5% VaR test of increasing window forecasting

	1%			5%		
	Ave. failure rate	Sig. Weibull test	Sig. DQ test	Ave. failure rate	Sig. Weibull test	Sig. DQ test
h=1						
HAR model	0.0354	SSEC, NSEI	All	0.0816	SSEC, NSEI	All
HAR-free model	0.0357	SSEC	All	0.0816	SSEC, NSEI	All
AR (22)-Lasso	0.0368	FTSE, SPX, N225, DAX, SSEC, NSEI	All	0.0856	All	All
AR (22)-adaLasso	0.0347	FTSE, SPX, N225, DAX, SSEC, NSEI	All	0.0845	All	All
AR (22)-grpLasso	0.0264	FTSE, N225, SSEC	All	0.0757	FTSE, N225, DAX, SSEC, NSEI, BSVP	All
AR (22)-ordLasso	0.0335	FTSE, SPX, N225, DAX, SSEC, NSEI	All	0.0825	All	All
AR (100)-Lasso	0.0367	FTSE, SPX, N225, DAX, SSEC, NSEI	All	0.0855	FTSE, N225, DAX, SSEC, NSEI, BSVP, MXX	All
AR (100)-adaLasso	0.0317	FTSE, N225, DAX, SSEC, NSEI	All	0.0804	FTSE, N225, DAX, SSEC, NSEI, MXX	All
AR (100)-grpLasso	0.0263	FTSE, SSEC	All	0.0750	N225, DAX, SSEC, NSEI, BSVP, MXX	All
AR (100)-ordLasso	0.0237	N225, DAX, SSEC, BSVP, MXX	All	0.0701	FTSE, N225, DAX, SSEC, NSEI	All
h=5						
HAR model	0.0016	FTSE, SPX, N225, NSEI	All	0.0048	FTSE, SPX, N225, DAX, NSEI, BSVP, MXX	All
HAR-Free model	0.0014	FTSE, SPX, N225, NSEI	All	0.0048	FTSE, SPX, N225, DAX, NSEI, BSVP, MXX	All
AR (22)-Lasso	0.0002	N225	All	0.0011	FTSE, N225, DAX, SSEC, NSEI, MXX	All
AR (22)-adaLasso	0.0001	N225	All	0.0011	FTSE, N225, NSEI, MXX	All
AR (22)-grpLasso	0.0001	N225	All	0.0009	FTSE, N225, MXX	All
AR (22)-ordLasso	0.0001	N225	All	0.0010	FTSE, SPX, N225, MXX	All
AR (100)-Lasso	0.0002	N225	All	0.0011	FTSE, N225, SSEC, MXX	All
AR (100)-adaLasso	0.0002	N225	All	0.0012	FTSE, N225, MXX	All
AR (100)-grpLasso	0.0001	N225	All	0.0010	FTSE, N225, SSEC, MXX	All
AR (100)-ordLasso	0.0001	None	All	0.0009	FTSE, SSEC, MXX	All
h=22						
HAR model	0.0000	None	All	0.0000	None	All
HAR-free model	0.0000	None	All	0.0000	None	All
AR (22)-Lasso	0.0000	None	All	0.0000	None	All
AR (22)-adaLasso	0.0000	None	All	0.0000	None	All
AR (22)-grpLasso	0.0000	None	All	0.0000	None	All
AR (22)-ordLasso	0.0000	None	All	0.0000	None	All
AR (100)-Lasso	0.0000	None	All	0.0000	None	All
AR (100)-adaLasso	0.0000	None	All	0.0000	None	All
AR (100)-grpLasso	0.0000	None	All	0.0000	None	All
AR (100)-ordLasso	0.0000	None	All	0.0000	None	All

Notes: this table provides the VaR results of increasing window test at the 1% and 5% VaR level. The average failure rate for each model over each index. The series are significant in the Weibull test and DQ test are listed.